MFDFA

Release 0.4.3

Aug 15, 2022

Contents

`MFDFA` is a python implementation of Multifractal Detrended Fluctuation Analysis, first developed by by Peng *et al.* [1] and later extended to study multifractality `MFDFA` by Kandelhardt *et al.* [2].

CHAPTER 1

Installation

`MFDFA` is available from PyPI, so you can use

```
pip install MFDFA
```

Then on your favourite editor just use

```
from MFDFA import MFDFA
```

Warning: To use the extension to include Empirical Mode Decomposition detrending you will also need

```
pip install EMD-signal
```

CHAPTER 2

An exemplary one-dimensional fractional Ornstein–Uhlenbeck process

For a more detailed explanation on how to integrate an Ornstein–Uhlenbeck process, see the kramersmoyal's package You can also follow the fOU.ipynb

2.1 Generating a fractional Ornstein–Uhlenbeck process

This is one method of generating a (fractional) Ornstein–Uhlenbeck process with $H = 0.7$, employing a simple Euler–Maruyama integration method

```python
# Imports
from MFDFA import MFDFA
from MFDFA import fgn
# where this second library is to generate fractional Gaussian noises

# integration time and time sampling
t_final = 500
delta_t = 0.001

# Some drift theta and diffusion sigma parameters
theta = 0.3
sigma = 0.1

# The time array of the trajectory
time = np.arange(0, t_final, delta_t)

# The fractional Gaussian noise
H = 0.7
dB = (t_final ** H) * fgn(N = time.size, H = H)

# Initialise the array y
y = np.zeros([time.size])

# Integrate the process
```

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```
for i in range(1, time.size):
   y[i] = y[i-1] - theta * y[i-1] * delta_t + sigma * dB[i]
```

And now you have a fractional process with a self-similarity exponent $H = 0.7$

2.2 Using the `MFDFA`

To now utilise the `MFDFA`, we take this exemplary process and run the (multifractal) detrended fluctuation analysis. For now lets consider only the monofractal case, so we need only $q = 2$.

```
# Select a band of lags, which usually ranges from
# very small segments of data, to very long ones, as
lag = np.unique(np.logspace(0.5, 3, 100, dtype=int))

# Notice these must be ints, since these will segment
# the data into chucks of lag size

# Select the power q
q = 2

# The order of the polynomial fitting
order = 1

# Obtain the (MF)DFA as
lag, dfa = MFDFA(y, lag = lag, q = q, order = order)
```

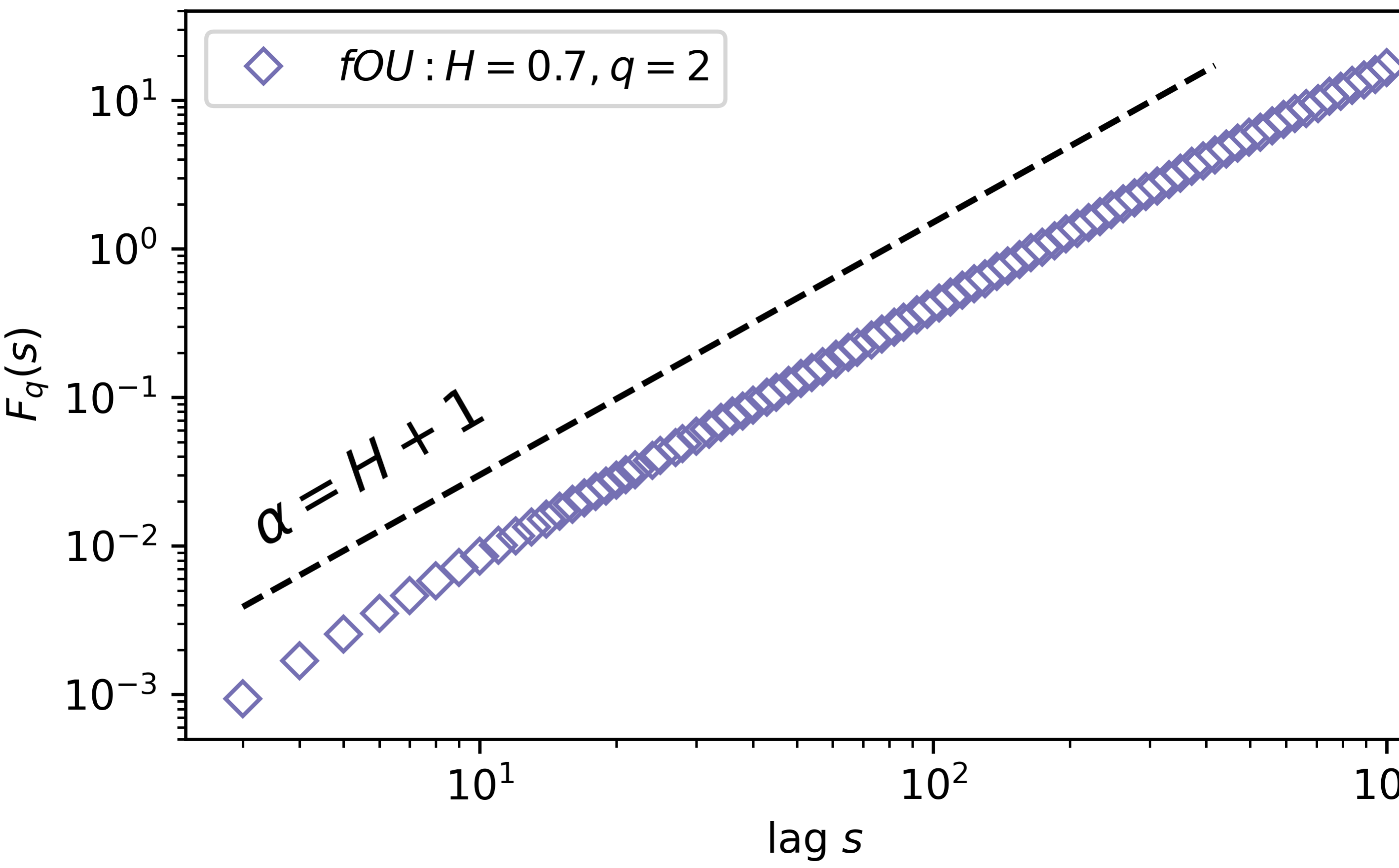
Now we need to visualise the results, which can be understood in a log-log scale. To find H we need to fit a line to the results in the log-log plot

```
# To uncover the Hurst index, lets get some log-log plots
plt.loglog(lag, dfa, 'o', label='fOU: MFDFA q=2')

# And now we need to fit the line to find the slope
# in a double logaritmic scales, i.e., you need to
# fit the logs of the results
H_hat = np.polyfit(np.log(lag)[4:20],np.log(dfa[4:20]),1)[0]

print('Estimated H = '+'{:.3f}'.format(H_hat[0]))


# Now what you should obtain is: slope = H + 1
```


$fOU: H = 0.7, q = 2$
$\alpha = H + 1$
$F_q(s)$
lag s
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^1
10^2

CHAPTER 3

Multifractality in one dimensional distributions

To show how multifractality can be studied, let us take a sample of random numbers of a symmetric Lévy distribution.

3.1 Univariate random numbers from a Lévy stable distribution

To obtain a sample of random numbers of Lévy stable distributions, use `scipy`'s `levy_stable`. In particular, take an α-stable distribution, with $\alpha = 1.5$

```
# Imports
from MFDFA import MFDFA
from scipy.stats import levy_stable

# Generate 100000 points
alpha = 1.5
X = levy_stable.rvs(alpha=alpha, beta = 0, size=10000)
```

For `MFDFA` to detect the multifractal spectrum of the data, we need to vary the parameter $q \in [-10, 10]$ and exclude 0. Let us also use a quadratic polynomial fitting by setting `order=2`

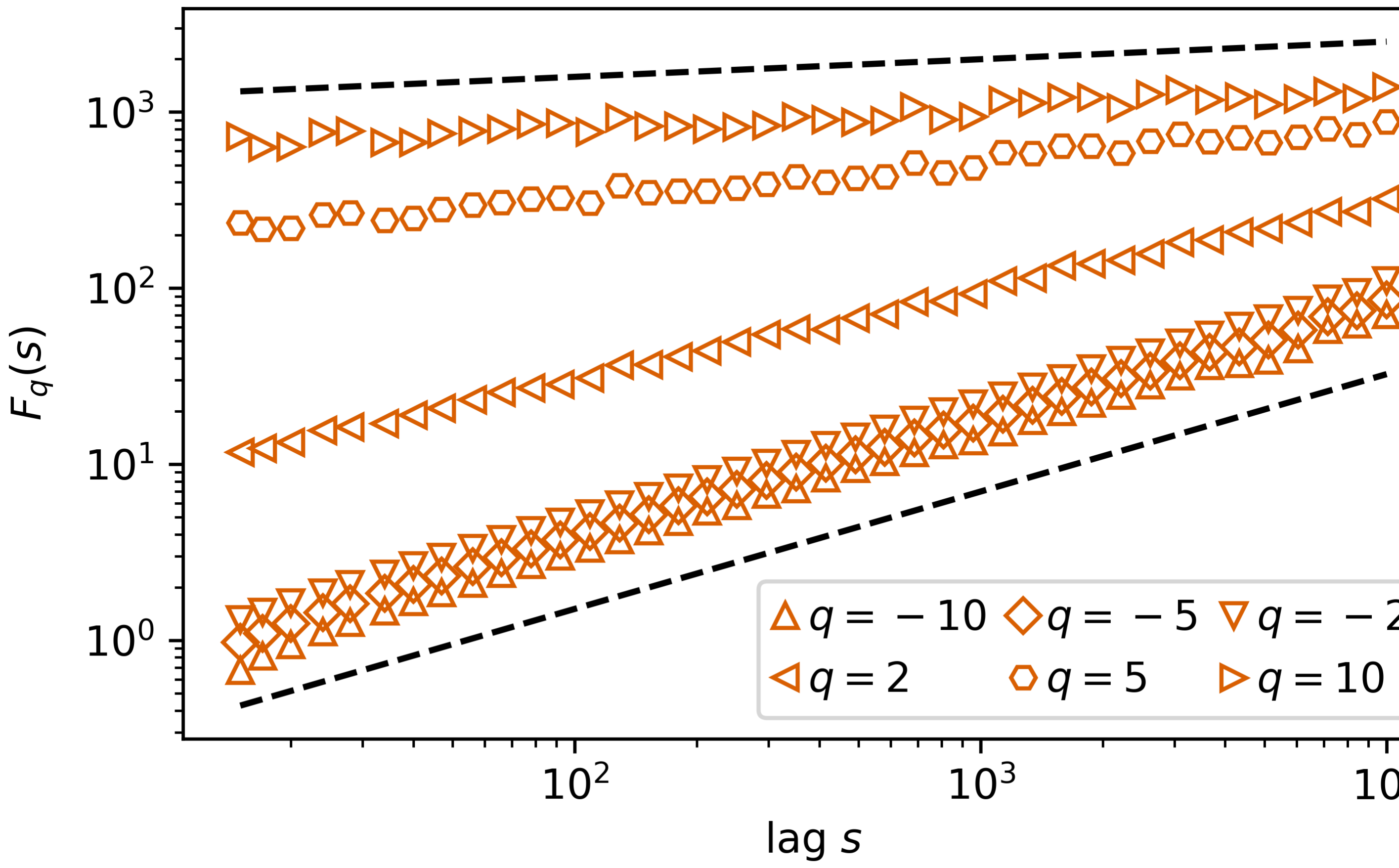
```
# Select a band of lags, which are ints
lag = np.unique(np.logspace(0.5, 3, 100).astype(int))

# Select a list of powers q
q_list = np.linspace(-10,10,41)
q_list = q_list[q_list!=0.0]

# The order of the polynomial fitting
order = 2

# Obtain the (MF)DFA as
lag, dfa = MFDFA(y, lag = lag, q = q_list, order = order)
```

Again, we plot this in a double logarithmic scale, but now we include 6 curves, from 6 selected $q = -10, -5-2, 2, 5, 10$. Include as well are the theoretical curves for $q = -10$, with a slope of $1/\alpha = 1/1.5$ and $q = 10$, with a slope of $1/q = 1/10$



CHAPTER 4

Extensions

MFDFA as seen since its development a set of enhancements. In particular the usage of Empirical Mode Decomposition as a source of detrending, instead of polynomial fittings, which allows for a more precise removal of known trends in the timeseries.

4.1 Employing Empirical Mode Decompositions for detrending

Empirical Mode Decomposition (EMD), or maybe more correctly described, the Hilbert–Huang transform is a transformation analogous to a Fourier or Hilbert transform that decomposes a one-dimensional timeseries or signal into its Intrinsic Mode Functions (IMFs). For our purposes, we simply want to employ EMD to detrend a timeseries.

Warning: To use this feature, you need to first install PyEMD (EMD-signal) with

```
pip install EMD-signal
```

4.1.1 Understanding `MFDFA`'s `EMD` detrender

Take a timeseries `y` and extract the Intrinsic Mode Functions (IMFs)

```python
# Import
from MFDFA import IMFs

# Extract the IMFs simply by employing
IMF = IMFs(y)
```

From here one obtains a `(..., y.size)`. Best now to study the different IMFs is to plot them and the timeseries `y`

```python
# Import
import matplotlib.pyplot as plt
```

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```
# Plot the timeseries and the IMFs 6,7, and 8
plt.plot(X, color='black')
plt.plot(np.sum(IMF[[6,7,8],:], axis=0).T)
```

4.1.2 Using `MFDFA` with `EMD`

To now perform the multifractal detrended fluctuation analysis, simply insert the IMFs desired to be subtracted from the timeseries. This will also for `order = 0`, not to do any polynomial detrending.

```
# Select a band of lags, which usually ranges from
# very small segments of data, to very long ones, as
lag = np.logspace(0.7, 4, 30).astype(int)

# Obtain the (MF)DFA by declaring the IMFs to subtract
# in a list in the dictionary of the extensions
lag, dfa = MFDFA(y, lag = lag, extensions = {"EMD": [6,7,8]})
```

4.2 Extended Detrended Fluctuation Analysis

In the publication Detrended fluctuation analysis of cerebrovascular responses to abrupt changes in peripheral arterial pressure in rats. the authors introduce a new metric similar to the conventional Detrended Fluctuation Analysis (DFA) which they denote *Extended* Detrended Fluctuation Analysis (eDFA), which relies on extracting the difference of the minima and maxima for each segmentation of the data, granting a new power-law exponent to study, i.e., as in eq. (5) in the paper

$$\mathrm{d}F(n) = \max[F(n)] - \min[F(n)],$$

which in turn results in

$$\mathrm{d}F(n) \sim n^{\beta}.$$

4.2.1 Using `MFDFA`'s `eDFA` extension

To obtain the `eDFA`, simply set the extension to `True` and add a new output function, here denoted `edfa`

```
# Select a band of lags, which usually ranges from
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lag, dfa, edfa = MFDFA(y, lag = lag, extensions = {'eDFA': True})
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4.3 Moving window for segmentation

For short timeseries the segmentation of the data—especially for large lags—results in bad statistics, e.g. if a timeseries has *2048* datapoints and one wishes to study the flucutation analysis up to a lag of *512*, only 4 segmentations of the data are possible for the lag *512*. Instead one can use an moving window over the timeseries to obtain better statistics at large lags.

4.3.1 Using `MFDFA`'s `window` extension

To utilise a moving window one has to declare the moving windows step-size, i.e., the number of data points the window will move over the data. Say we wish to increase the statistics of the aforementioned example to include a moving window moving *32* steps (so one has 64 segments at a lag of *512*)

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# Obtain the (MF)DFA by declaring the IMFs to subtract
# in a list in the dictionary of the extensions
lag, dfa, edfa = MFDFA(y, lag = lag, extensions = {'window': 32})
```

CHAPTER 5

Literature

[1] Peng, C.-K., Buldyrev, S. V., Havlin, S., Simons, M., Stanley, H. E., & Goldberger, A. L. (1994). *Mosaic organization of DNA nucleotides*. Physical Review E, 49(2), 1685–1689

[2] Kantelhardt, J. W., Zschiegner, S. A., Koscielny-Bunde, E., Havlin, S., Bunde, A., & Stanley, H. E. (2002). *Multifractal detrended fluctuation analysis of nonstationary time series*. Physica A: Statistical Mechanics and Its Applications, 316(1-4), 87–114

CHAPTER 6

Funding

Helmholtz Association Initiative Energy System 2050 - A Contribution of the Research Field Energy and the grant No. VH-NG-1025, STORM - Stochastics for Time-Space Risk Models project of the Research Council of Norway (RCN) No. 274410, and the E-ON Stipendienfonds.

CHAPTER 7

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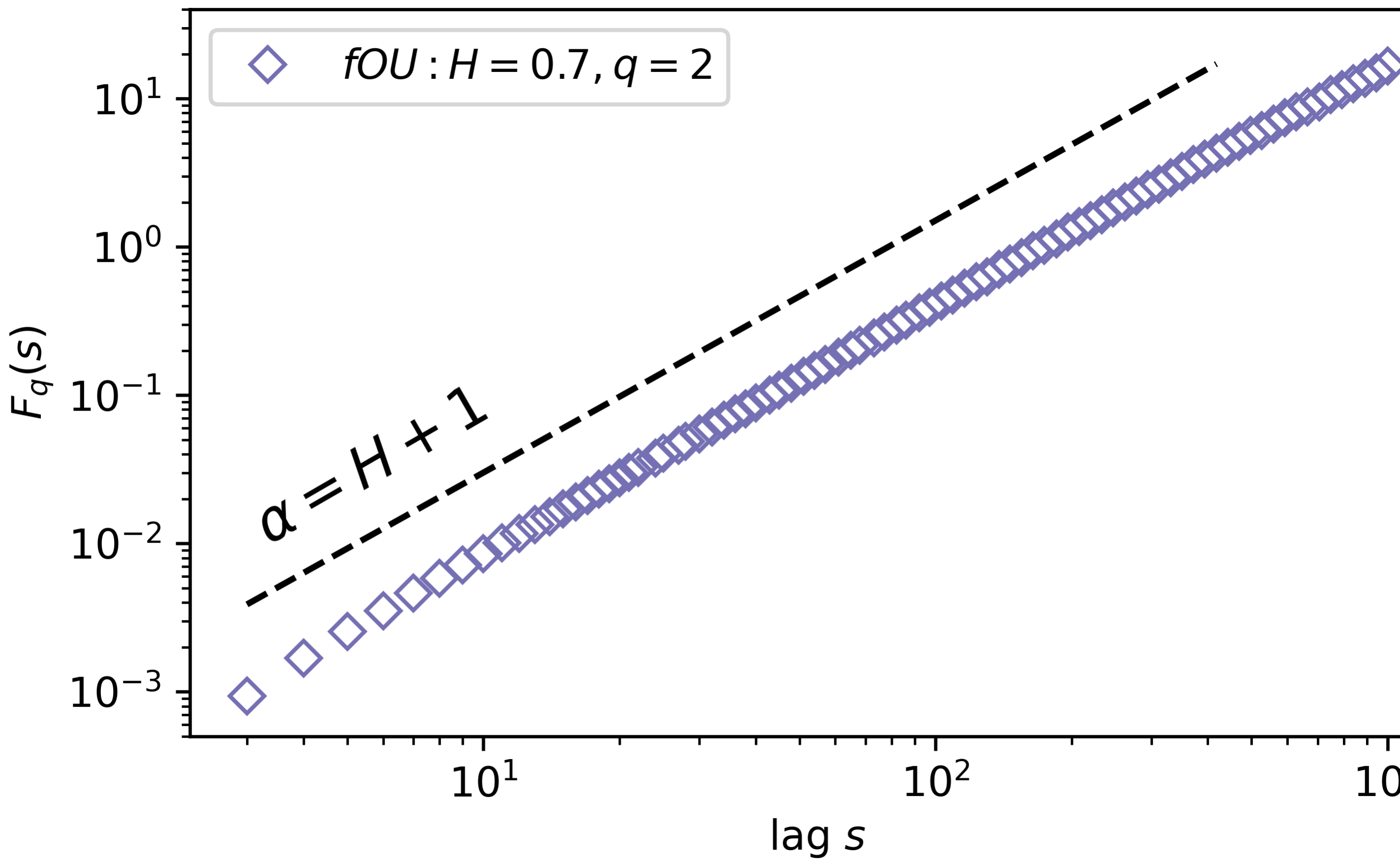

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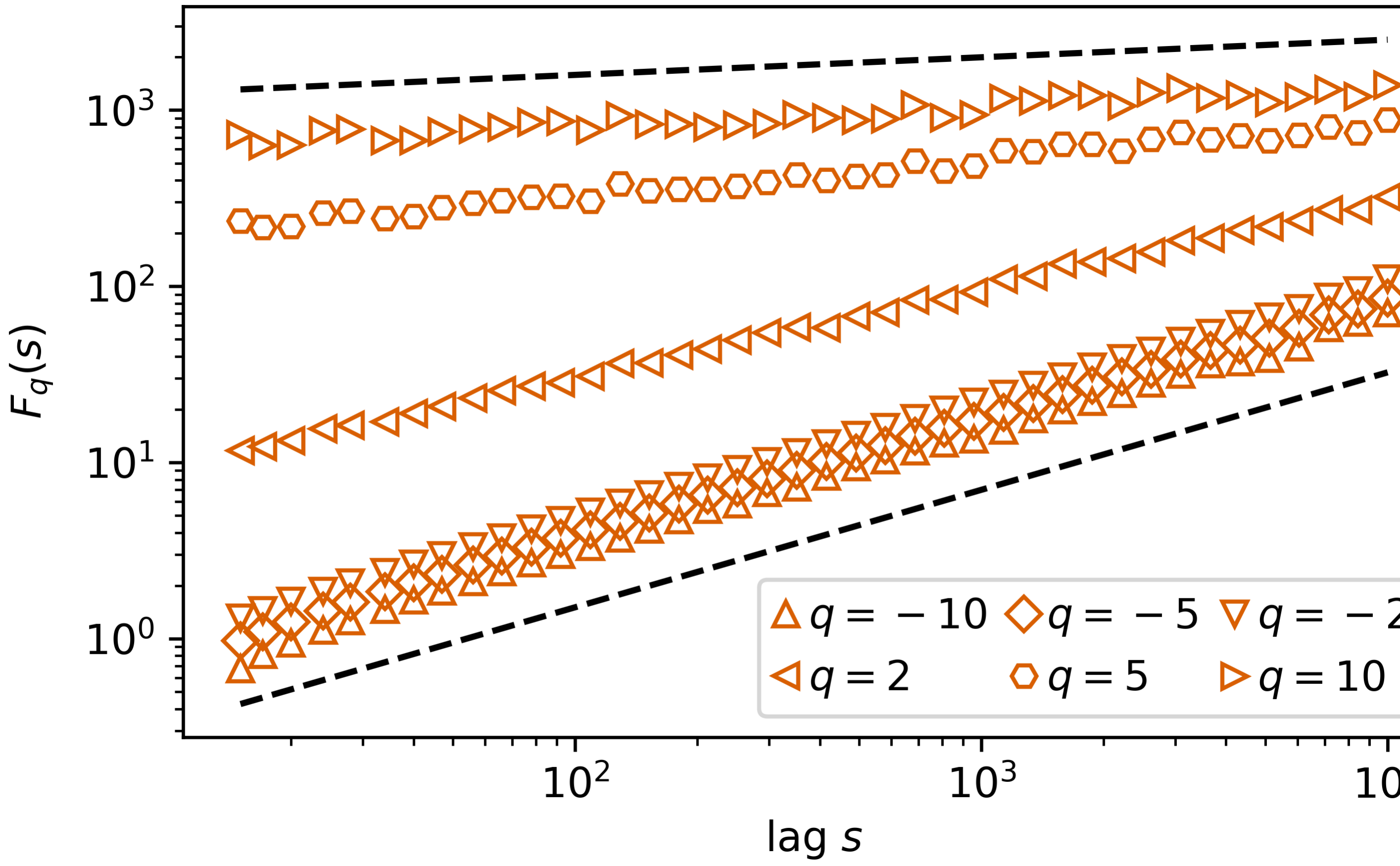

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7.4.3 Moving window for segmentation

For short timeseries the segmentation of the data—especially for large lags—results in bad statistics, e.g. if a timeseries has *2048* datapoints and one wishes to study the flucutation analysis up to a lag of *512*, only 4 segmentations of the data are possible for the lag *512*. Instead one can use an moving window over the timeseries to obtain better statistics at large lags.

Using `MFDFA`'s `window` extension

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```python
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# in a list in the dictionary of the extensions
lag, dfa, edfa = MFDFA(y, lag = lag, extensions = {'window': 32})
```

7.5 Functions

7.5.1 MFDFA

`MFDFA.MFDFA.`**`MFDFA`**(*timeseries: numpy.ndarray*, *lag: numpy.ndarray*, *order: int = 1*, *q: numpy.ndarray = 2*, *stat: bool = False*, *modified: bool = False*, *extensions: dict = {'EMD': False, 'eDFA': False, 'window': False}*) → Tuple[numpy.array, numpy.ndarray]

Multifractal Detrended Fluctuation Analysis of timeseries. MFDFA generates a fluctuation function F^2(q,s), with s the segment size and q the q-powers, Take a timeseries X, find the integral Y = cumsum(X), and segment the timeseries into N segments of size s.

$$F^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}[Y_{(v-1)s+i} - y_{v,i}]^2, \text{ for } v = 1, 2, \dots, N_s,$$

with $y_{v,i}$ the polynomial fittings of order m. Having obtained the variances of each (detrended) segment, average over s and increase s, to obtain the fluctuation function $F_q^2(s)$ depending on the segment length.

$$F_q^2(s) = \left\{\frac{1}{N_s}\sum_{v=1}^{N_s}[F^2(v,s)]^{q/2}\right\}^{1/q}$$

The fluctuation function $F_q^2(s)$ can now be plotted in a log-log scale, the slope of the fluctuation function $F_q^2(s)$ vs the s-segment size is the self-similarity scaling $h(q)$

$$F_q^2(s) \sim s^{h(q)}.$$

If $H0$ in a monofractal series, use a second integration step by setting `modified = True`.

Parameters

- **`timeseries`** (*`np.ndarray`*) – A 1-dimensional timeseries *(N, 1)*. The timeseries of length *N*.
- **`lag`** (*`np.ndarray of ints`*) – An array with the window sizes to calculate (ints). Notice *min(lag) > order + 1* given a polynomial fit of order *m* needs at least *m* points. The results are meaningless for 'order = m' and for lag > size of data / 4 since there is low statistics with < 4 windows to divide the timeseries.
- **`order`** (int (default *1*)) – The order of the polynomials to approximate. *order = 1* is the DFA1, which is a least-square fit of the data with a first order polynomial (a line), *order = 2* is a second-order polynomial, etc.. *order = 0* skips the detrending process and hence gives the non-detrended fluctuation functions, i.e., simply Fluctuation Analysis.
- **`q`** (np.ndarray (default *2*)) – Fractal exponent to calculate. Array in *[-10,10]*. The values = 0 will be removed, since the code does not converge there. *q = 2* is the standard Detrended Fluctuation Analysis as is set a default.
- **`stat`** (bool (default *False*)) – Calculates the standard deviation associated with each segment's averaging.
- **`modified`** (bool (default *False*)) – For data with the Hurst index 0, i.e., strongly anticorrelated, a standard MFDFA will result in inacurate results, thus a further integration of the timeseries yields a modified scaling coefficient.
- **`extensions`** (*`dict`*) –
  - ***EMD*: list (default *False*)** If not *None*, requires a list of indices of the user-chosen IMFs obtained from an (externally performed) EMD analysis. The indexing starts from *0*. Will enforce *order = 0* since there is no need for a polynomial detrending.

  - ***eDFA*: bool (default *False*)** A method to evaluate the strength of multifractality. Calls function *eDFA()*.
  - ***window*: bool (default *False*)** A moving window for smaller timeseries. Set *window* as int > 0 with the number of steps the window shoud move over the data. *window = 1* will move window by *1* step. Since the timeseries is segmented at each lag lenght, any window choise > lag is only segmented once.

Returns

- **lag** (*np.ndarray of ints*) – Array of lags, realigned, preserving only different lags and with entries > order + 1
- **f** (*np.ndarray*) – A array of shape *(size(lag),size(q))* of variances over the indicated lag windows and the indicated q-fractal powers.

References

`MFDFA.MFDFA.`**`eDFA`**(*F: numpy.ndarray*) → numpy.ndarray

In the reference indicated below a measure of nonstationarity was added by including a subsequent calculation of the extrema of the DFA. Denoted $dF_q^2(s)$ the difference of the extrema at each segment, i.e.,

$$dF_q^2(s) = \max[F_q^2(s)] - \min[F_q^2(s)]$$

Parameters **`F`** (*`np.ndarray`*) – Fluctuation function given by the *MFDFA()*.

Returns **res** – Difference of *max* and *min*.

Return type np.ndarray

Notes

New in version 0.3.

References

7.5.2 Empirical Mode Decomposition for detrending timeseries

`MFDFA.emddetrender.`**`detrendedtimeseries`**(*timeseries: numpy.ndarray*, *modes: list*) → numpy.ndarray

The function calculates the Intrinsic Mode Functions (IMFs) of a given timeseries, subtracts the user-chosen IMFs for detrending, and returns the detrended timeseries. Based on based on Dawid Laszuk's PyEMD found at https://github.com/laszukdawid/PyEMD

Parameters

- **`timeseries`** (*`np.ndarray`*) – A 1-dimensional timeseries of length *N*.
- **`modes`** (*`list`*) – List of integers indicating the indices of the IMFs to be subtracted/detrended from the *timeseries*.

Returns **detrendedTimeseries** – Detrended 1-dimensional *timeseries*.

Return type np.ndarray

Warning: To use the extension Empirical Mode Decomposition for detrending, the `pyEMD` library is needed.

```
pip install EMD-signal
```

Notes

New in version 0.3.

References

`MFDFA.emddetrender.`**`IMFs`**(*timeseries: numpy.ndarray*) → numpy.ndarray

Extract the Intrinsic Mode Functions (IMFs) of a given timeseries.

Parameters **`timeseries`** (*`np.ndarray`*) – A 1-dimensional timeseries of length *N*.

Notes

New in version 0.3.

Returns **IMFs** – The Intrinsic Mode Functions (IMFs) of the Empirical Mode Decomposition. These are of shape *(. . . , timeseries.size)*, with the first dimension varying depending on the data. Last entry is the residuals.

Return type np.ndarray

7.5.3 Fractional Gaussian noise

`MFDFA.fgn.`**`fgn`**(*N: int*, *H: float*) → numpy.ndarray

Generates fractional Gaussian noise with a Hurst index H in (0,1). If H = 1/2 this is simply Gaussian noise. The current method employed is the Davies–Harte method, which fails for H 0. A Cholesky decomposition method and the Hosking's method will be implemented in later versions.

Parameters

- **`N`** (*`int`*) – Size of fractional Gaussian noise to generate.
- **`H`** (*`float`*) – Hurst exponent H in (0,1).

Returns **f** – A array of size N of fractional Gaussian noise with a Hurst index H.

Return type np.ndarray

7.6 License

7.7 Contact

If you need help with something, find a bug, issue, or typo on the repository or in the code, you can contact me here: leonardo.rydin@gmail.com or open an issue on the GitHub repository.

Bibliography

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